Influence of Pile Diameter on Constant of Horizontal Subgrade Reaction n_h

Ming-min WANG & Ting-hao MAO
Ove Arup & Partners Hong Kong Ltd

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ABSTRACT

The constant of horizontal subgrade reaction (n_h) is a cornerstone parameter in the Winkler-based subgrade reaction method for analyzing laterally loaded piles. This article re-examines whether n_h is independent of pile diameter. The previous assumption that the deflection of the pile is linearly proportional to the dimension of a pressure bulb does not accurately reflect the real situation. Three-dimensional finite element method was employed to conduct a series of analyses, demonstrating that for the same soil material, n_h increases almost linearly with pile diameter.

By comparing the deflection results of pile obtained from Plaxis3D, with those derived from the subgrade reaction method using n_h, it is evident that when employing the n_h method, the influence of pile diameter should be considered to reduce material costs and carbon emissions in projects involving laterally loaded piles.

1 INTRODUCTION

The analysis of laterally loaded piles is critical for infrastructure such as bridge abutments, pile-supported retaining walls, and high-rise buildings. The subgrade reaction method, first conceptualized by Winkler (1867), models soil-pile interaction using a series of discrete springs along the pile shaft. In this model, the soil resistance per unit length (P_h) is expressed as $P_h = n_h \cdot z \cdot \delta_h$, where z represents the depth below ground level and δ_h represents the lateral deflection. Current standards (HK GEO Publication No.1/2006; HK Code of Practice for Foundations 2017) prescribe n_h as a function of soil type and stress history but treat it to be independent of pile geometry. For granular soils, the value of the constants of horizontal sub-grade reaction (n_h) are provided in Table 5.1 of the HK Code of Practice for Foundations, 2017.

Table 1 Correlation of Constant of Horizontal Subgrade Reaction with SPT N-values for Granular Soil (HK Code of Practice for Foundations 2017)

SPT N-value	n_h for dry or moist sand ($kN/m^2/m$)	n_h for submerged sand ($kN/m^2/m$)
4 to 10	2200	1300
11 to 30	6600	4400
31 to 50	17600	10700

Based on the current method, since n_h is not considered as a function of pile dimensions, a pile with a very large diameter would theoretically exhibit the same soil reaction per unit length as a pile with a very small diameter. In other words, increasing the pile diameter does not enhance the soil reaction acting on the pile. When calculating the deflection of a pile under horizontal loading, the influence of pile diameter (D) on the flexural stiffness of the pile body is taken into account, while the influence of pile diameter (D) on soil resistance per unit length (P_h) is neglected. This discrepancy may lead to an underestimation of deflection for small-diameter piles and an overestimation for large-diameter piles.

2 LITERATURE REVIEW

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The Winkler model, despite its simplicity, underpins modern lateral pile analysis due to its computational efficiency. Terzaghi (1955) first adapted it for soils, proposing n_h values for clays and sands. Reese (1974) advanced the method via *p*-*y* curves, but retained n_h as depth-linear and diameter-agnostic.

Randolph (1981) identified limitations in the Winkler approach, notably its neglect of soil continuity and stress redistribution. Ashour et al. (1998) demonstrated that n_h could vary with pile flexibility (*EI*), but diameter effects remained unexplored. Hong Kong GEO Report No. 21 (1992) hinted that the approach of using n_h for wall analysis has been questioned because the behavior of a pile is governed by its width.

For soil-pile interaction, Dodds (2005) studied the pile behavior in large pile groups under lateral loading, provided insight into the mechanics of large pile group lateral stiffness, various issues such as installation effects, pile, pile head and soil conditions. Wang (2015, 2022 & 2023) studied soil-pile interaction and loads distribution of different kinds of pile; it is found that the diameter influence is significant.

There are many researchers using centrifuge tests or loading test of trial pile to study the behavior of pile taking lateral loads. Russo & Viggiani (2008) reviewed the behavior of piles under lateral loading based on full scale and centrifuge tests results. Nadilla & Prakoso (2019) studied the correlation between the subgrade reaction modulus and the soil N-SPT value is examined by conducting numerical analyses of 34 pile cyclic lateral load tests in Jakarta. Chin, Sew & Chung (2009) presented the results and interpretation of a lateral load test on a fully instrumented spun pile. Li, Wei, Feng & Chen (2019) conducted a series of field tests to examine the behavior of pile foundation subjected to adjacent surcharge loading in deep soft soils.

Sensitive study results are also provided by some scholars. Gebremichael & Berg (2022) used both LPile and Plaxis3D to study horizontally loaded piles. Zhao & Wang (2018) analyzed the pile lateral response from deflection measurement data with a compressive sampling-based method. Tommy, Widjaja & Hutabarat (2023) used three-dimensional finite element method and *P-y* Curve to study the lateral bearing capacity of single bored pile. Law & Cheng (2015) studied the *P-delta* effects of piles embedded in cohesionless soil.

3 METHODOLOGY

3.1 Theoretical adjustment of n_h method

For the traditional subgrade reaction method, Terzaghi assumed that deflection was linearly proportional to the dimension of a pressure bulb and, consequently, linearly proportional to pile width. Hence, for a pile with width d_1 applying a pressure *q* per unit area and an associated deflection y_1, and another pile of width d_2 ($=nd_1$) applying the same pressure per unit area with an associated deflection y_2, it is assumed that y_2 should equal ny_1. Based on this assumption, the following relationship applies:

$$(k_h)_2 = \frac{qd_2}{y_2} = \frac{q \cdot nd_1}{ny_1} = \frac{qd_1}{y_1} = (k_h)_1 \qquad (1)$$

Where $(k_h)_1$ = subgrade modulus for pile of width d_1, and

$(k_h)_2$ = subgrade modulus for pile of width d_2.

Thus, the subgrade modulus for a pile has traditionally been considered independent of the pile diameter.

However, the assumption that y_2 equals ny_1 is questionable. Consider a scenario where q_1=1 kPa, d_1=1 m, y_1=10 mm, and d_2=2 m, implying n=2. According to the definition prior to equation (1), $y_2=ny_1$=20mm. Nevertheless, under the same ground conditions and pressure (q_2=1 kPa), the deflection y_2 typically does not reach 20 mm.

Furthermore, the deflection of a pile should not be simplistically assumed to be linearly proportional to the dimension of a pressure bulb, as the size of the pressure bulb cannot quantitatively reflect the stress variation.

To investigate the relationship between the subgrade modulus for a pile and the pile diameter, we employed the three-dimensional finite element method to conduct a series of analyses.

3.2 Decoupling the dimension effects on pile flexural rigidity and soil reaction

Due to the fact that both pile flexural rigidity and soil reaction may be influenced by pile diameter simultaneously, in order to investigate the effect of pile diameter on n_h, it is necessary to maintain the pile stiffness constant. Assuming a pile subjected to a uniformly distributed lateral load along its length, if the pile diameter is increased while keeping the load unchanged, the pile displacement will be affected solely by the variation in soil reaction. This approach effectively eliminates the influence of pile flexural rigidity.

Plaxis3D was employed to preform simulations of a series of piles subjected to uniformly distributed lateral loads. Based on the results from these pile (referred to as pile series A), the soil springs represented by soil resistance per unit length (P_h) were obtained. These soil springs were then incorporated into the GSA model to simulate another series of piles (referred to as pile series B), which are subjected to point loads at the pile head. Additionally, Plaxis3D was also used to simulate pile series B for comparison purpose.

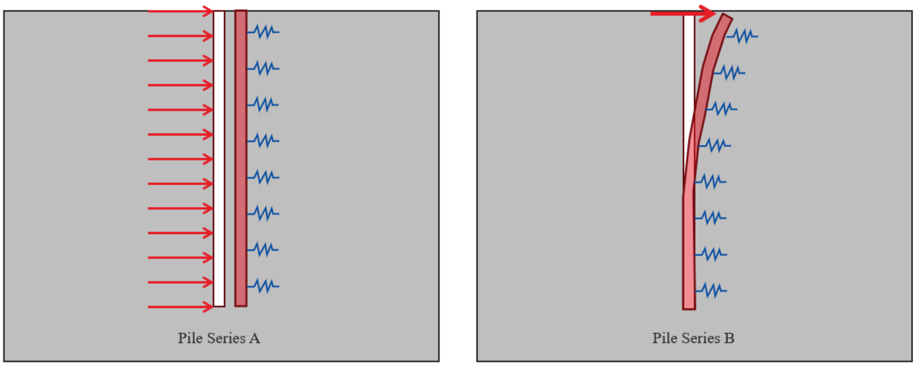

Figure 1: Decoupling the dimension effects on pile flexural rigidity and soil reaction

3.3 Numerical Modeling Framework

Two series of three-dimensional finite element models were developed in Plaxis3D to simulate pile series A and pile series B. These models consisted of 30 m long reinforced concrete piles with diameters ranging from 0.6 m to 3.0 m (in increments of 0.2 m) embedded in homogeneous sand.

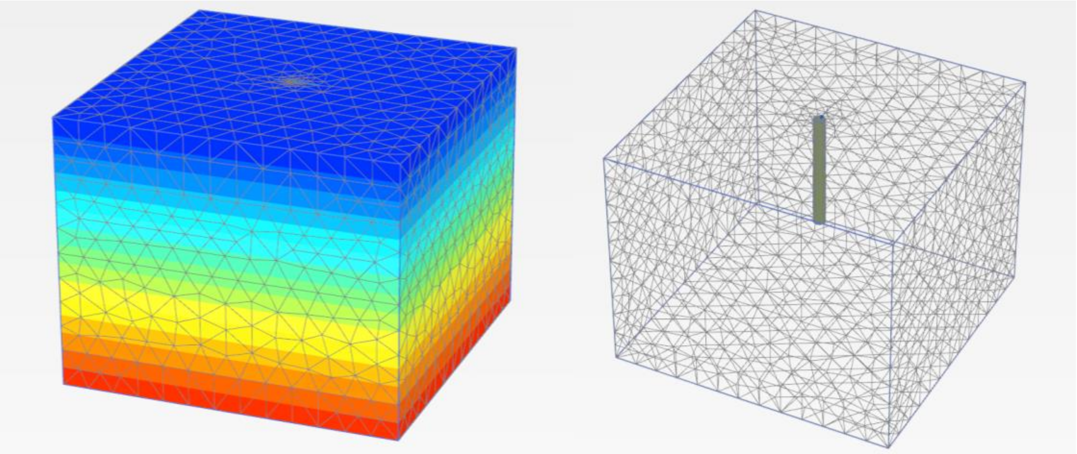

Figure 2: Plaxis3D model of pile series A

The transverse, longitudinal and vertical directions of the models are 75 m, 75 m, and 90 m, respectively. The piles were modeled using an isotropic linear elastic material model with an elastic modulus of 31.5 GPa, a Poisson’s ratio of 0.2, and a density of 2500 kg/m^3.

The Mohr-Coulomb criterion was used for the soil. Based on the trial test results reported by Li (2019), the soil has a density of 1939 kg/m^3, an elastic modulus of 78.8 MPa, a Poisson’s ratio of 0.25, a cohesion of 0.1 kPa and an internal friction angle of 15.0°. The element sizes were refined to 0.1D near the pile and gradually increased to 0.5D at the domain boundaries. Lateral displacements were constrained at the domain edges, and the base was fully fixed.

A uniformly distributed lateral load of 20 kN/m along the pile length was applied to pile series A. Monotonic lateral point loads of 200 kN were applied at the pile head of pile series B.

Back-calculation was performed to derive n_h from the Plaxis3D results of pile series A. n_h was calculated as follows:

$$n_h = \frac{P_h}{z \cdot \delta_h} \qquad (2)$$

4 INFLUENCE OF PILE DIAMETER ON n_h

4.1 Numerical Results of Pile Series A

Fig. 3 shows the displacement of Pile Series A. As can be observed from Fig.3, for piles with diameters ranging from 0.6 m to 1.4 m, the upper potion (approximately 5 m below the ground surface) exhibits larger deflections than expected. This is due to the reduced confinement and lower yield strength of the soil near the ground surface (Fig. 4). The relatively smaller deflections in the lower portion (approximately 5 m above the pile toe) can be attributed to the three-dimensional confinement at the pile toe and the frictional resistance at the bottom surface of the pile. Wider piles tend to reduce localized yielding by distributing strain over a larger area, thereby preserving more elastic soil behavior.

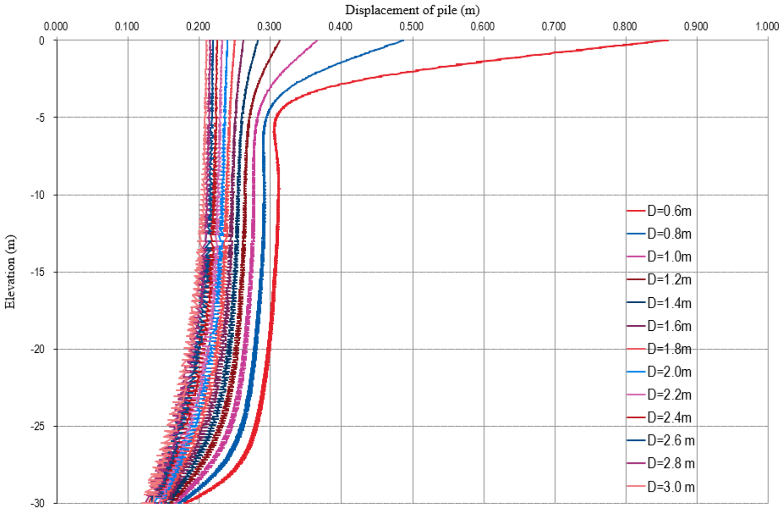

Figure 3: Displacement of pile series A

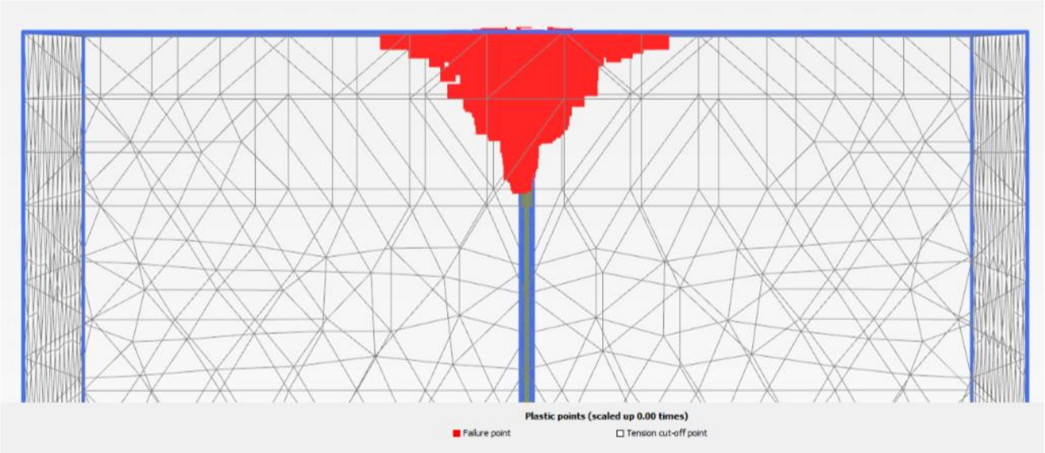

Figure 4: Plastic point of soil in Plaxis3D model

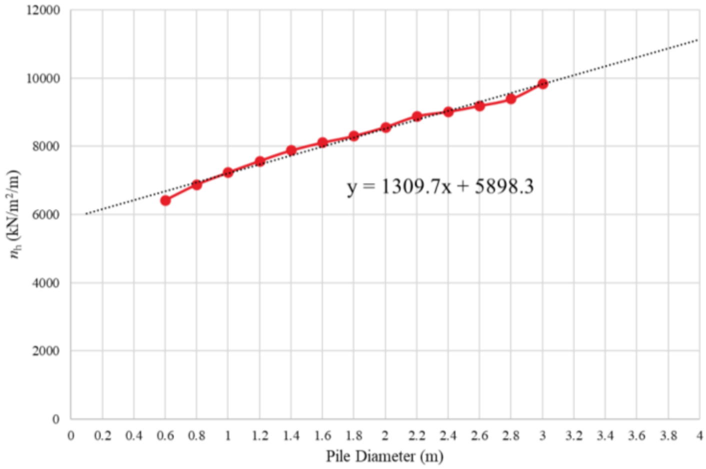

Figure 5: Relation between n_h (10 m below ground surface) and pile diameter D

From Fig. 5 we can observe that for the same soil material, n_h of 10 m below ground surface increased from 6400 kN/m^2/m to 9850 kN/m^2/m when pile diameter D changes from 0.6 m to 3.0 m. The trend followed:

$$n_h = 1309.7D + 5898.3 \qquad (3)$$

Larger diameters generated broader passive wedges and deeper shear strain penetration, enhancing confinement and soil stiffness.

4.2 Deflection Results of Pile Series B based on n_h of Code of Practice for Foundations 2017

Since the Young's modulus for the soil in the Plaxis3D model can be converted into SPT N-value, the correlation of constant of horizontal subgrade reaction and SPT N-values for granular soil presented in Table 1 can be utilized to calculate the soil spring. Subsequently, Pile Series B was simulated using both Plaxis3D and GSA (n_h method). A point load of 600 kN was applied to each pile head.

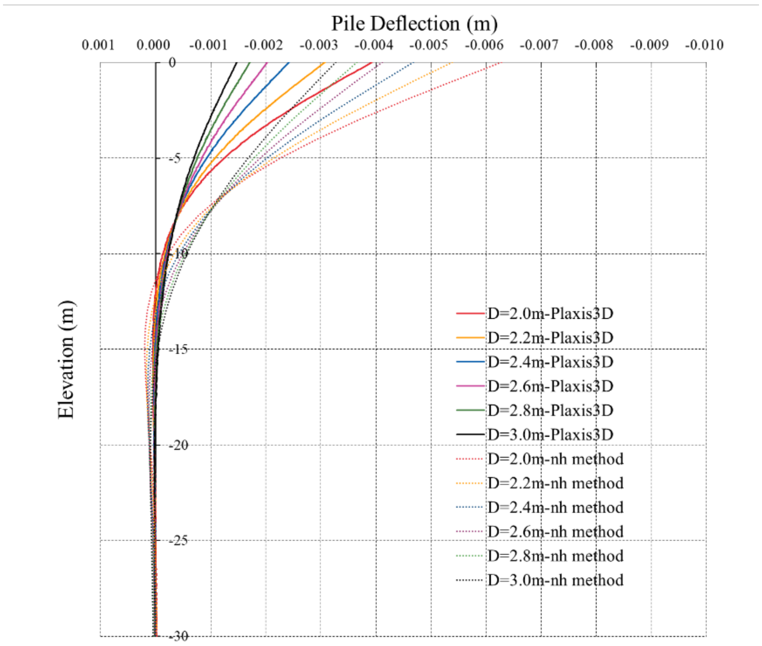

Figure 6: Deflection of Pile Series B (D ranges from 2.0 m to 3.0 m)

From Fig. 6, it can be observed that the pile deflections of Pile Series B calculated by GSA using the n_h method are larger than those calculated by Plaxis3D. However, upon comparing Fig. 5 and Fig. 6, it is evident that when the pile diameter is relatively small, the pile deflections of Pile Series B calculated by GSA using the n_h method are not consistently larger than those calculated by Plaxis3D. This discrepancy arises because the soil in front of the pile has yielded in the Plaxis3D simulations.

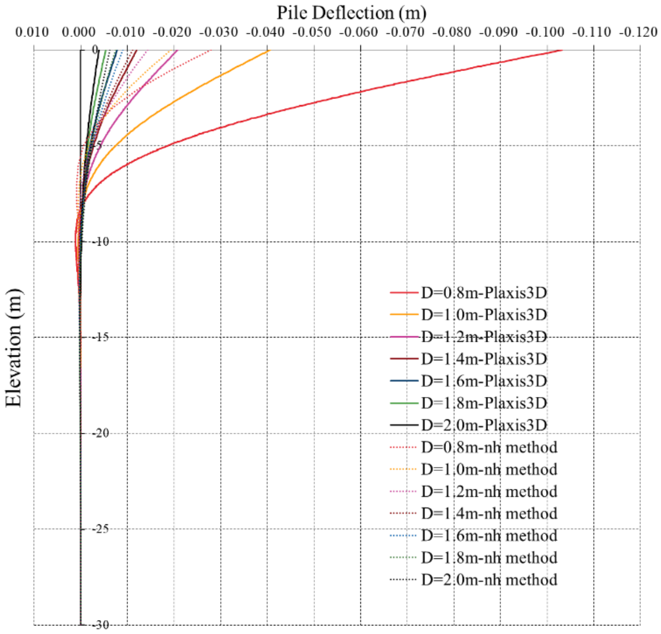

Figure 7: Deflection of Pile Series B (*D* ranges from 0.8 m to 2.0 m)

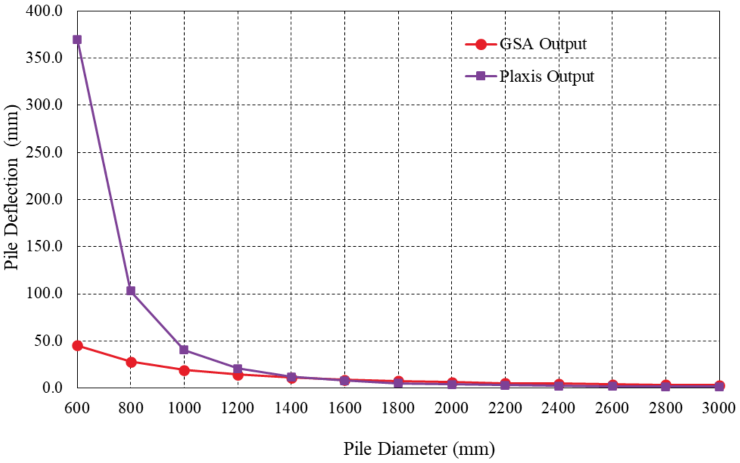

Figure 8: Maximum pile deflection

Table 2 Deflection Results of Pile Series B

Pile Diameter *D (m)*	Plaxis3D Deflection (mm)	n_h method Deflection (mm)	Difference Ratio
0.8	103.2	28.0	-72.9%
1.0	40.7	19.4	-52.4%
1.2	20.8	14.4	-31.0%
1.4	12.1	11.2	-7.5%
1.6	7.9	9.0	14.6%
1.8	5.4	7.4	38.1%
2.0	3.9	6.3	59.9%
2.2	3.1	5.4	75.0%
2.4	2.4	4.7	93.2%
2.6	2.0	4.1	102.6%
2.8	1.7	3.7	113.3%
3.0	1.5	3.3	122.0%

From Fig. 8 and Table 2, it can be observed that when pile diameter is less than 1.4 m, the maximum deflection obtained from Plaxis3D is significantly larger than the pile deflections calculated by GSA using the n_h method. This discrepancy arises because the soil in front of the pile has yielded in the Plaxis3D simulations, whereas the n_h method does not account for the nonlinear behavior of the soil.

When the pile diameter exceeds 1.6m, the difference ratio between the maximum deflection obtained from Plaxis3D and the pile deflections calculated by GSA using the n_h method increases substantially with increasing pile diameter. This occurs because the influence of pile diameter (D) on soil resistance per unit length (P_h) is neglected in the n_h method. Consequently, this leads to inaccuracies in predicting lateral deflections for large-diameter piles.

5 Suitable pile diameters for n_h method

According to the Plaxis3D simulation results, it can be observed that the n_h method is not suitable for all pile diameters.

For piles with small diameters subjected to relatively large lateral loads, the soil in front of the pile will yield. Since the traditional n_h method does not account for the nonlinear behavior of soil springs, when the soil yields, the deflection results based on the n_h method will be significantly underestimated.

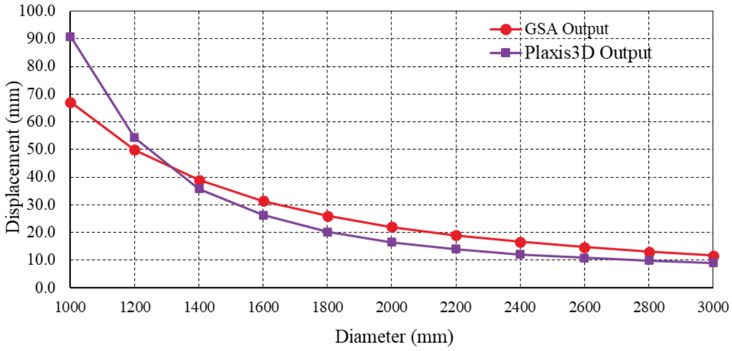

Figure 9: Pile deflection with different diameter when n_h = 1300 kN/m^2/m

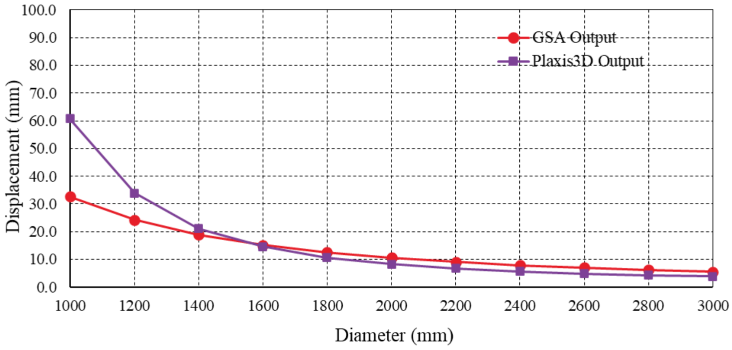

Figure 10: Pile deflection with different diameter when n_h = 4400 kN/m^2/m

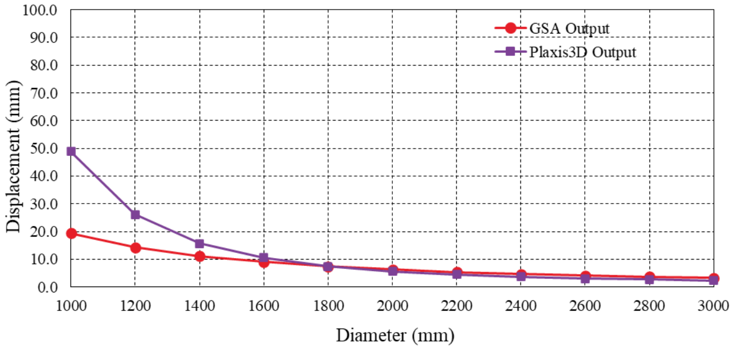

Figure 11: Pile deflection with different diameter when n_h = 10700 kN/m^2/m

Fig. 9 to 11 indicate that there is always a crossover point between the curve of Plaxis3D results and the curve derived from the n_h method. Furthermore, for relatively small pile diameters, the pile deflections obtained from Plaxis3D are larger than those calculated by the n_h method, while for relatively large pile diameters, the pile deflections obtained from Plaxis3D are consistently smaller than those calculated by the n_h method.

Since the pile itself has structural capacity, piles with small diameters typically cannot sustain relatively large lateral loads. However, the limitation imposed by structural capacity may not correspond to the appropriate boundary for the n_h method. Therefore, designers should verify whether the n_h method can be applied based on whether the soil has yielded, rather than whether the pile structure has failed.

6 PRACTICAL RECOMMENDATIONS

Based on the findings in section 5, it can be observed that the n_h method is not suitable for piles with relatively small diameters while taking large lateral loads. When the soil in front of the pile does not yield, the n_h method can be applied but requires adjustment by considering the pile diameter.

To obtain more accurate deflection results for Pile Series B, the value of the constant of horizontal subgrade reaction can be adjusted as follows:

$$n_h' = (D - D_0)n_h \quad (4)$$

Where n_h' is the revised value of the constant of horizontal sub-grade reaction,

D_0 is the baseline diameter for n_h, corresponding to the crossover point in Fig. 8 for this study.

Table 3 and Fig. 11 present the revised deflection results for pile series B. The different ratio between n_h methods and Plaxis3D results are significantly reduced.

Table 3 Revised Deflection Results of Pile Series B

Pile Diameter D (m)	Plaxis3D Deflection (mm)	Revised n_h Method Deflection (mm)	Difference Ratio
0.8	103.2	49.0	——
1	40.7	27.2	——
1.2	20.8	16.8	——
1.4	12.1	11.2	-7.44%
1.6	7.9	7.9	-0.32%
1.8	5.4	5.7	5.06%
2	3.9	4.3	11.46%
2.2	3.1	3.4	9.27%
2.4	2.4	2.7	12.60%
2.6	2.0	2.2	8.81%
2.8	1.7	1.8	7.27%
3	1.5	1.5	1.20%

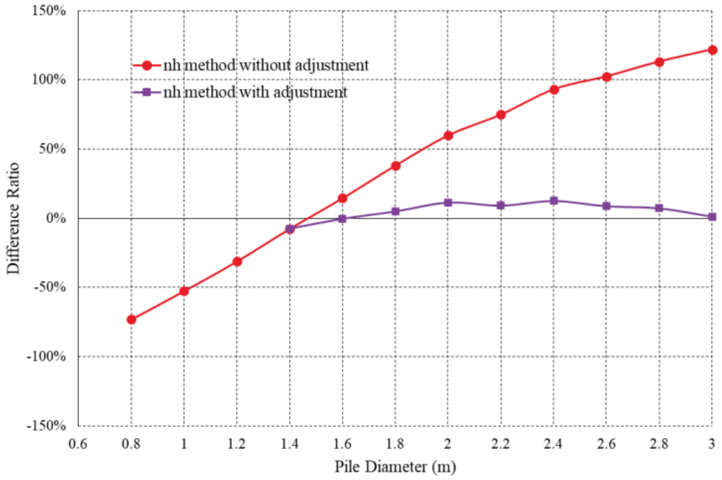

Figure 12: Difference Ratio of nh methods

It can be observed that the subgrade modulus for a pile is dependent on the pile diameter. For large-diameter piles, this adjustment can lead to safe and more economical designs. This finding will help reduce material costs and carbon emissions for projects involving piles subjected to lateral loads.

7 CONCLUSIONS

The constant of horizontal subgrade reaction (n_h) is a cornerstone parameter in the Winkler-based subgrade reaction method for analyzing laterally loaded piles. This article revisits whether the subgrade modulus for a pile is independent of the pile diameter.

Conventional design frameworks assume n_h to be depth-dependent but independent of pile dimensions; however, the author found that the assumption of linearly proportionality between pile deflection and the dimension of a pressure bulb does not reflect the real situation.

To investigate the relationship between the subgrade modulus for a pile and pile diameter, a series of analyses were conducted using the three-dimensional finite element method. By decoupling the dimension effects on pile flexural rigidity and soil reaction, it was observed that for the same soil material, n_h increases almost linearly with pile diameter.

By comparing the deflection results of piles obtained from Plaxis3D and those calculated using subgrade reaction method based on n_h of Hong Kong Code of Practice for Foundations (2017), it can be seen that for different types of sand, the pile deflections obtained from Plaxis3D are larger than those calculated by the n_h method for relatively small pile diameters, while the pile deflections obtained from Plaxis3D are consistenly smaller than those calculated by the n_h method for relatively large pile diameters.

Since the pile possess structural capacity, piles with small diameter typically cannot sustain relatively large lateral loads, meaning most previous projects do not have safety risks. However, the limitation imposed by structural capacity may not the same value of the appropriate boundary of the n_h method. Therefore, designers should verify whether the n_h method can be applied based on whether the soil has yielded, rather than whether the pile structure has failed.

When the soil in front of the pile does not yield, the n_h method can be applied but should ideally be adjusted by considering the influence of pile diameter. For large-diameter piles, this adjustment can provide safe and more economical designs. This finding will help reduce material costs and carbon emissions for projects involving piles subjected to lateral loads.

It should be noted that this study focused on homogeneous ground profiles; layered systems require further analysis. The effectiveness of the n_h method under cyclic or dynamic loads remains unexplored. Since the geological profiles for real projects are far more complex than the Plaxis3D or GSA models mentioned in this article, trial piles and lateral loading tests are recommended to accurately assess the performance of piles subjected to lateral loads.

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